We are concerned with the run-up/run-down problem for the non-linear shallow water wave equation. In 1958 Carrier and Greenspan were able to linearize this problem for the case of a plane sloping beach. Their method has received much attention and is now referred to as the Carrier-Greenspan transform. In the 1990s Pelinovsky et al. generalized it to consider parabolic bathymetry with constant slope. The main feature of these two situations is that the Carrier-Greenspan transform can be performed explicitly, producing analytical solutions to the run-up problem for these cases.

This transform was recently extended by Rybkin-Pelinovsky-Didenkulova to inclined bathymetries of arbitrary cross sections. By using this transform, the original non-linear water wave equations are reduced to a linear Klein-Gordon equation on a half-line. The resulting linear system does not, in general, have a d'Alembert solution. Thus numerical methods are required.

We consider a sloping bathymetry of finite length with u shaped cross section. For the offshore boundary, we assume that the wave height is 0. This model can physically represents a constantly sloping beach with u shaped cross sections that has a wall some distance from the shore or a wave traveling in a constantly sloping beach with u shaped cross sections that has a fixed point at some distance from the shore. Some physical examples of such bays include some glaciers and bays similar to Lituya bay.

To solve the run-up problem for such bays, the generalized Carrier-Greenspan transform is applied to the physical system and then a spectral method is used to find a series solution for the resulting linear system. This series then in turn is transformed into physical coordinates via an inverse transformation. No analytical solution was found, so a simi-analytic numerical algorithm was developed.

This talk will discuss the generalized Carrier-Greenspan transform and how it was used to transform our non-linear run-up problem into a linear problem, how a series solution to the corresponding linear system found, the difficulties that had to be overcome to build a numerical algorithm that uses the analytic series solution as well as the back conversion to the physical system and the advantages/limitations that out numerical method has.